SMALL OSCILLATIONS OF A GAS-FILLED SPHERICAL CHAMBER IN VISCOELASTIC POLYMER MEDIA

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A number of special features are characteristic for the mechanical behavior of polymer materials; on the phenomenological level they find their description within the framework of complicated continuous models [1-3]. Among those special features there are various kinds of viscoelasticity which must be taken into account when analyzing the flow and deformation of polymers. In the present article small oscillations are investigated of a gas-filled spherical chamber in a viscoelastic polymer medium described by a rheological equation with eight constants [3]. An exact solution is obtained of the equation of small oscillations of the chamber, the effect being studied of the rheological parameters of the medium on the oscillations. The analyzed problem is of particular interest in connection with the problems of acoustic cavitation in aqueous solutions of polymers [4-6].

The rheological equation with eight constants [3] which is used to describe the mechanical behavior of a number of polymer media is given by

$$\tau_{ik}+\lambda_1\frac{D\tau_{ik}}{Dt}+\mu_0\tau_{jj}e_{ik}-\mu_1(\tau_{ij}e_{jk}+\tau_{jk}e_{ij})+\nu_1\tau_{jn}e_{jn}\delta_{ik}=2\eta_0\left(e_{ik}+\lambda_2\frac{De_{ik}}{Dt}-2\mu_2e_{ij}e_{jk}+\nu_2e_{jn}e_{jn}\delta_{ik}\right)$$
$$\tau_{ik}=p_{ik}+p\delta_{ik},\qquad e_{ik}=\frac{1}{2}\left(\frac{\partial v_k}{\partial x_i}+\frac{\partial v_i}{\partial x_k}\right),\quad \omega_{ik}=\frac{1}{2}\left(\frac{\partial v_k}{\partial x_i}-\frac{\partial v_i}{\partial x_k}\right) \tag{1}$$
$$\frac{D\tau_{ik}}{Dt}=\frac{\partial\tau_{ik}}{\partial t}+v_j\frac{\partial\tau_{ik}}{\partial x_j}+\omega_{ij}\tau_{jk}+\omega_{kj}\tau_{ij}$$

In the above p_{ik}, e_{ik}, and ω_{ik} are the stress tensor, tensor of the deformation rates, and the whirl tensor respectively; p is the isotropic pressure; v_i are the projections of the velocity vector on the coordinate axes x_i; λ_1, λ_2, ρ_0, ρ_1, ρ_2, η_0, ν_1, ν_2 are the rheological constants; D/Dt is the Jauman derivative.

Small radial oscillations are considered of a gas-filled spherical chamber of radius R immersed in an unbounded incompressible viscoelastic medium (1) with density ρ. Spherical coordinates r, θ, φ are introduced with the origin at the center of the chamber. Assuming that the flow has spherical symmetry one finds from the continuity equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2v_r)=0 \tag{2}$$

the equations

$$v_r=R^2\dot{R}r^{-2},\qquad \dot{R}=dR/dt \tag{3}$$

The projection on the r axis of the equation of motion of the continuous medium in terms of stresses is

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}\right)=-\frac{\partial p}{\partial r}+\frac{\partial\tau_{rr}}{\partial r}+\frac{2(\tau_{rr}-\tau_{\varphi\varphi})}{r} \tag{4}$$

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In the above one has taken into account that $\tau_{\varphi\varphi} = \tau_{\theta\theta}$ holds by virtue of the previously assumed symmetry. Integrating Eq. (4) with respect to r from R to ∞, using Eq. (3) and proceeding to another variable $y = 1/3\,(r^3 - R^3)$, the equation of motion is obtained of the boundary of the chamber, namely

$$\rho\,(R\ddot{R} + {}^3/_2\dot{R}^2) = p\,(R) - p\,(\infty) + \tau_{rr}\,(\infty) - \tau_{rr}\,(R) + 2\int_0^\infty \frac{\tau_{rr} - \tau_{\varphi\varphi}}{3y + R^3}\,dy \qquad (5)$$

In Eq. (5) $\tau_{rr}(R)$, $p(R)$ and $\tau_{rr}(\infty)$, $p(\infty)$ denote the stress and pressure on the surface of the chamber or at infinity.

For p(R) [4] one has the following boundary condition:

$$p\,(R) = p_1\,(R) - 2\sigma R^{-1} + \tau_{rr}\,(R), \quad p_1\,(R) = p_{10}\,(R_0 R^{-1})^{3k} \qquad (6)$$

where $p_1(R)$ and p_{10} are pressures of gas inside the chamber at any time instant and at the initial instant respectively, R_0 is the initial radius of the chamber, σ is the coefficient of surface tension, k is the index of the polytropic curve. The quantity p_{10} can be found from the equilibrium condition for the chamber under pressure p_∞ at the initial instant,

$$p_{10} = p_\infty + 2\sigma R_0^{-1} \qquad (7)$$

The equations for τ_{rr} and $\tau_{\varphi\varphi}$ are now written down. By substituting Eq. (3) in Eq. (1) and changing to the variable y one obtains

$$\lambda_1 \frac{\partial\tau_{rr}}{\partial t} + \tau_{rr} + (4\mu_1 - 2\mu_0 - 2\nu_1)\frac{R^2\dot{R}}{3y + R^3}\tau_{rr} + (2\nu_1 - 4\mu_0)\frac{R^2\dot{R}}{3y + R^3}\tau_{\varphi\varphi} =$$
$$-4\eta_1\left[\frac{R^2\dot{R}}{3y + R^3} + \lambda_2\frac{2R\dot{R}^2 + R^2\ddot{R}}{3y + R^3} + (4\mu_2 - 3\nu_2 - 3\lambda_2)\frac{R^4\dot{R}^2}{(3y + R^3)^2}\right]$$
$$\lambda_1\frac{\partial\tau_{\varphi\varphi}}{\partial t} + \tau_{\varphi\varphi} + (2\mu_0 - 2\mu_1 + 2\nu_1)\frac{R^2\dot{R}}{3y + R^3}\tau_{\varphi\varphi} + \qquad (8)$$
$$+ (\mu_0 - 2\nu_1)\frac{R^2\dot{R}}{3y + R^3}\tau_{rr} = 2\eta_1\left[\frac{R^2\dot{R}}{3y + R^3} + \lambda_2\frac{2R\dot{R}^2 + R^2\ddot{R}}{3y + R^3} - (2\mu_2 - 6\nu_2 + 3\lambda_2)\frac{R^4\dot{R}^2}{(3y + R^3)^2}\right]$$

The quantity $p(\infty)$ is represented as

$$p\,(\infty) = p_\infty - p_0 \sin\,\omega t \qquad (9)$$

where $p_0/p_\infty \ll 1$. Under periodical pressure small radial oscillations close to the equilibrium position begin to appear in the chamber. By setting $R = R_0 + \Delta R$, where ΔR denotes a small deviation from the quantity R_0, Eqs. (5) and (8) become linearized. This yields the following system of linear equations for the unknowns ΔR and τ_{rr}:

$$\rho R_0\frac{d^2(\Delta R)}{dt^2} + \frac{3k}{R_0}\left[p_\infty + \frac{2\sigma}{R_0}\left(1 - \frac{1}{3k}\right)\right]\Delta R = p_0 \sin\omega t + 3\int_0^\infty \frac{\tau_{rr}}{3y + R_0^3}\,dy, \qquad \tau_{\varphi\varphi} = -\frac{1}{2}\tau_{rr} \qquad (10)$$

$$\lambda_1\frac{\partial\tau_{rr}}{\partial t} + \tau_{rr} = -\frac{4\eta_0 R_0^2}{3y + R_0^3}\left(\frac{d\,(\Delta R)}{dt} + \lambda_2\frac{d^2(\Delta R)}{dt^2}\right) \qquad (11)$$

The initial conditions are given by

$$\Delta R = d\,(\Delta R)\,/\,dt = \tau_{rr} = 0 \quad \text{for} \quad t = 0$$

The system (10) and (11) can be solved by using operational calculus [7]. By taking the Laplace transforms of the equations one obtains an equation for the transform of ΔR,

$$\Delta R^* = \gamma\,(\lambda_1^{-1} + s)\,[(s^2 + \omega^2)\,(s - s_1)\,(s - s_2)\,(s - s_3)]^{-1} \qquad (12)$$

In the above s is the complex Laplace-transformation variable; s_1, s_2, s_3 are roots of the cubic equation

$$s^3 + as^2 + bs + c = 0$$
$$a = \lambda_1^{-1} + 2\alpha\lambda_2\lambda_1^{-1}, \quad b = \beta + 2\alpha\lambda_1^{-1}, \quad c = \beta\lambda_1^{-1}$$
$$\alpha = 2\eta_0\,(\rho R_0^2)^{-1}, \quad \gamma = p_0\omega\,(\rho R_0)^{-1}$$

$$\beta = 3k\,[p_\infty + 2\sigma R_0^{-1}\,(1 - (3k)^{-1})]\,(\rho R_0^2)^{-1} \tag{13}$$

Using Cardan's formulas one can write the solutions of Eq. (13) as

$$\begin{aligned} & s_1 = A + B - a/3, \quad s_{2,3} = -\tfrac{1}{2}(A+B) \pm i\sqrt{3}\,(A-B)/ \\ & /2 - a/3 \\ & A = \sqrt[3]{-u/2 + \sqrt{Q}}, \quad B = \sqrt[3]{-u/2 - \sqrt{Q}}, \quad Q = (\vartheta/3)^3 + (u/2)^2 \\ & u = 2\,(a/3)^3 - ab/3 + c, \quad v = -\tfrac{1}{3}a^2 + b \end{aligned} \tag{14}$$

Applying now the inverse Laplace transformation to the relation (12) one finds an expression for ΔR. The following three cases take place depending on the sign of Q.

1. $Q > 0$. Equation (13) has only one real root (s_1) and two complex-conjugate roots (s_2 and s_3). By representing the $s_{2,3}$ in the form $s_{2,3} = \delta \pm i\mu$, one obtains

$$\begin{aligned} \Delta R &= \gamma\omega^{-1} D \sin(\omega t + \alpha_1) + \frac{\gamma e^{\delta t}\sin(\mu t + \alpha_2)}{\mu\,[4\delta^2\mu^2 + (\delta^2 - \mu^2 + \omega^2)^2]^{1/2}} + \\ &+ \frac{\gamma(\lambda_1^{-1} + s_1)\,e^{\delta t}\sin(\mu t + \alpha_3)}{\mu\,[\mu^2(3\delta^2 - \mu^2 + \omega^2 - 2\delta s_1)^2 + [(\delta - s_1)(\delta^2 + \omega^2 - \mu^2) - 2\delta\mu^2]^2]^{1/2}} + \frac{\gamma(\lambda_1^{-1} + s_1)\,e^{s_1 t}}{(s_1^2 + \omega^2)(s_1^2 - 2\delta s_1 + \delta^2 + \mu^2)} \\ D &= \left\{\frac{[\omega(\omega^2 - b) + (s_1 + \lambda_1^{-1})\,2\omega\delta]^2 + [(a\omega^2 - c) + (\lambda_1^{-1} + s_1)(\omega^2 - \delta^2 - \mu^2)]^2}{[4\omega^2\delta^2 + (\omega^2 - \delta^2 - \mu^2)^2]\,[\omega^2(\omega^2 - b)^2 + (a\omega^2 - c)^2]}\right\}^{1/2} \\ \operatorname{tg}\alpha_1 &= -\frac{2\omega\delta\,[\omega^2(\omega^2 - b)^2 + (a\omega^2 - c)^2] + (s_1 + \lambda_1^{-1})\,\omega(\omega^2 - b)\,[4\omega^2\delta^2 + (\omega^2 - \delta^2 - \mu^2)^2]}{(\omega^2 - \delta^2 - \mu^2)[\omega^2(\omega^2 - b)^2 + (a\omega^2 - c)^2] + (s_1 + \lambda_1^{-1})(a\omega^2 - c)[4\omega^2\delta^2 + (\omega^2 - \delta^2 - \mu^2)^2]} \\ \operatorname{tg}\alpha_2 &= \frac{2\delta\mu}{\mu^2 - \omega^2 - \delta^2}, \quad \operatorname{tg}\alpha_3 = \frac{\mu(3\delta^2 - \mu^2 + \omega^2 + 2\delta s_1)}{(s_1 - \delta)(\delta^2 - \mu^2 + \omega^2) + 2\delta\mu^2} \end{aligned} \tag{15}$$

2. $Q = 0$. Equation (13) possesses three real roots, two of them being the same. The formula for ΔR now becomes

$$\begin{aligned} \Delta R &= \gamma\omega^{-1} D \sin(\omega t + \alpha_1) + \frac{\gamma(s_1 + \lambda_1^{-1})}{(s_2 - s_1)^2(s_1^2 + \omega^2)}\,e^{s_1 t} + \\ &+ \frac{\gamma(\lambda_1^{-1} + s_1)[2s_2(s_1 - s_2) - (s_2^2 + \omega^2)] - 2\gamma s_2(s_2 - s_1)^2}{(s_2^2 + \omega^2)^2(s_2 - s_1)^2}\,e^{s_2 t} + \frac{\gamma(\lambda_1^{-1} + s_2)}{(s_2^2 + \omega^2)(s_2 - s_1)}\,t e^{s_2 t} \\ D &= (s_1^2 + \omega^2)^{-1/2}(s_2^2 + \omega^2)^{-1}(s_2 - s_1)^{-2}\{(\lambda_1^{-1} + s_2)^2(s_1^2 + \omega^2) \times \\ &\times (s_2 - s_1)^2 - 2(\lambda_1^{-1} + s_1)(\lambda_1^{-1} + s_2)(s_2 - s_1)[s_1(s_2^2 - \omega^2) + 2\omega^2 s_2] + (\lambda_1^{-1} + s_1)^2(s_2^2 + \omega^2)^2\}^{1/2} \\ \operatorname{tg}\alpha_1 &= \frac{2\omega s_2(s_2 - s_1)(s_1^2 + \omega^2)(\lambda_1^{-1} + s_2) - \omega(\lambda_1^{-1} + s_1)(s_2^2 + \omega^2)^2}{(s_2^2 - \omega^2)(s_2 - s_1)(s_1^2 + \omega^2)(\lambda_1^{-1} + s_2) - s_1(\lambda_1^{-1} + s_1)(s_2^2 + \omega^2)^2} \end{aligned} \tag{16}$$

3. $Q < 0$. Equation (13) possesses three different real roots. In this case one has

$$\begin{aligned} \Delta R &= \gamma\omega^{-1} D \sin(\omega t + \alpha_1) + \frac{\gamma(\lambda_1^{-1} + s_1)}{(s_1^2 + \omega^2)(s_1 - s_2)(s_1 - s_3)}\,e^{s_1 t} + \frac{\gamma(\lambda_1^{-1} + s_2)}{(s_2^2 + \omega^2)(s_2 - s_1)(s_2 - s_3)}\,e^{s_2 t} + \frac{\gamma(\lambda_1^{-1} + s_3)}{(s_3^2 + \omega^2)(s_3 - s_1)(s_3 - s_2)}\,e^{s_3 t} \\ D &= \left\{\frac{[\omega(\omega^2 - b) + (\lambda_1^{-1} + s_1)\,\omega(s_2 + s_3)]^2 + [(a\omega^2 - c) + (\lambda_1^{-1} + s_1)(\omega^2 - s_2 s_3)]^2}{[\omega^2(s_2 + s_3)^2 + (\omega^2 - s_2 s_3)^2]\,[\omega^2(\omega^2 - b)^2 + (a\omega^2 - c)^2]}\right\}^{1/2} \\ \operatorname{tg}\alpha_1 &= -\frac{\omega(s_2 + s_3)[\omega^2(\omega^2 - b)^2 + (a\omega^2 - c)^2] + (\lambda_1^{-1} + s_1)\,\omega(\omega^2 - b)[\omega^2(s_2 + s_3)^2 + (\omega^2 - s_2 s_3)^2]}{(\omega^2 - s_2 s_3)[\omega^2(\omega^2 - b)^2 + (a\omega^2 - c)^2] + (\lambda_1^{-1} + s_1)(a\omega^2 - c)[\omega^2(s_2 + s_3)^2 + (\omega^2 - s_2 s_3)^2]} \end{aligned} \tag{17}$$

The relations (15)-(17) represent all possible solutions to our problem. It is not difficult to find that in all three cases one has $\mathrm{Re}\{s_i\} \le 0$ ($i = 1, 2, 3$). Therefore if $t \to \infty$ the term $\gamma\omega^{-1} D \sin(\omega t + \alpha_1)$ is the only one left in the formulas (15)-(17), and describes the forced oscillations of the chamber with frequency ω. The amplitude of the oscillation is given by the quantity D.

Two limiting cases, $\lambda_1 \to 0$ and $\lambda_1 \to \infty$ are now studied in detail.

For $\lambda_1 = 0$ Eq. (11) describes a fluid which needs some time to get into motion produced by a suddenly applied force [8]. The condition $Q > 0$ for such a medium indicates that $\beta(1 + 2\alpha\lambda_2) > \alpha^2$. The formula (15) now becomes

$$\begin{aligned} \Delta R &= \gamma M\,[\omega^{-1}\sin(\omega t + \psi_1) + \varkappa^{-1} e^{-\varepsilon_1 t}\sin(\varkappa t + \psi_2)] \\ M &= [4\alpha^2\omega^2 + (\beta - \omega^2 z)^2]^{-1/2}, \quad \varepsilon_1 = \alpha z^{-1}, \quad z = 1 + 2\alpha\lambda_2 \end{aligned}$$

$$\operatorname{tg}\psi_1 = \frac{2\varkappa\omega}{\omega^2 z - \beta}, \quad \operatorname{tg}\psi_2 = \frac{2\varkappa(\beta z - \alpha^2)^{1/2}}{2\varkappa^2 + \omega^2 z^2 - \beta z}, \quad \varkappa = z^{-1}(\beta z - \alpha^2)^{1/2} \tag{18}$$

The resonance frequency ω_p is now determined from the extremality of the amplitude M. One obtains

$$\omega_p = z^{-1}(\beta z - 2\alpha^2)^{1/2} \tag{19}$$

The resonance is only observed if $\beta z > 2\alpha^2$. In the case $\alpha^2 < \beta z \le 2\alpha^2$ the maximal amplitude occurs formally for $\omega = 0$. One now investigates the effect of retardation time λ_2 on the oscillations. For $\lambda_2 = 0$ the formula (19) gives an expression for the resonance frequency ω_p° in a Newtonian fluid. If the inequality $\beta > 4\alpha^2$ is true, then for any $\lambda_2 \neq 0$ $\omega_p < \omega_p^\circ$ and for λ_2 increasing, the resonance frequency decreases monotonically. If, however, $\beta < 4\alpha^2$, then for λ_2 increasing from 0 to $(2\alpha)^{-1}(4\alpha^2\beta^{-1}-1)$, the quantity ω_p increases from ω_p° to $1/4\sqrt{2\beta}\alpha^{-1}$, and then it again declines monotonically to zero. In the resonance case the oscillation amplitude is $M_p = (2\alpha\varkappa)^{-1}$. With λ_2 increasing the quantity M_p increases. The damping coefficient is determined by ε_1; it declines with λ_2 increasing. It is noted that the damping coefficient in a viscoelastic fluid is always smaller than in a Newtonian fluid. Finally, it follows from Eq. (18) that with λ_2 increasing the frequency of the eigenoscillations of the chamber is reduced; the phase shift between chamber oscillations increases as well as the pressure at infinity.

In the case of Q = 0, proceeding to the limit for $\lambda_1 \to 0$ in the formula (16) results in

$$\Delta R = \gamma z(\alpha^2 + \omega^2 z^2)^{-1}[\omega^{-1}\sin(\omega t + \psi_3) + te^{-\varepsilon_1 t} + 2\alpha z(\alpha^2 + \omega^2 z^2)^{-1}e^{-\varepsilon_1 t}], \quad \operatorname{tg}\psi_3 = 2\alpha\omega z(\omega^2 z^2 - \alpha^2)^{-1} \tag{20}$$

It follows from Eq. (20) that for a given frequency ω the maximal amplitude of the chamber-forced oscillations takes place in the medium for $\lambda_2 = (2\alpha)^{-1}(\alpha\omega^{-1}-1)$. If $\alpha\omega^{-1} \le 1$, the oscillation amplitude depends monotonically on the parameter λ_2; the amplitude is then maximal if $\lambda_2 = 0$, that is, for a Newtonian fluid.

By proceeding to the limit in the formula (17) with $\lambda_1 \to 0$ one obtains an expression for ΔR in the case of Q < 0, namely

$$\Delta R = \gamma\omega^{-1}M\sin(\omega t + \psi_1) + {}^1/_2\gamma(\alpha^2 - \beta z)^{-1/2}e^{-\varepsilon_2 t}[(\varepsilon_2^2 + \omega^2)^{-1} - (\varepsilon_3^2 + \omega^2)^{-1}e^{(\varepsilon_2 - \varepsilon_3)t}], \quad \varepsilon_{2,3} = z^{-1}[\alpha \mp (\alpha^2 - \beta z)^{1/2}] \tag{21}$$

Since for Q < 0 the condition $\lambda_1 = 0$ takes the form $\beta z < \alpha^2$, the formula (19) cannot be employed for the resonance frequency. The maximal amplitude takes place formally for $\omega = 0$.

The second limiting case is now considered. With λ_1 and η_0 tending to infinity one assumes also that $\eta_0/\lambda_1 \to G$. Then Eq. (11) describes an incompressible viscoelastic body of the Voigt model [9]. The condition Q > 0 now becomes $\beta + 2E > \lambda_2^2 E^2$, where $E = 2G(\rho R_0^2)^{-1}$. From the relation (15) one finds that

$$\begin{gathered}\Delta R = \gamma N[\omega^{-1}\sin(\omega t + \varphi_1) + q^{-1}e^{-n_1 t}\sin(qt + \varphi_2)] \\ N = [4\omega^2 n_1^2 + (d - \omega^2)^2]^{-1/2}, \quad d = \beta + 2E, \quad n_1 = \lambda_2 E \\ q = (d - n_1^2)^{1/2}, \quad \operatorname{tg}\varphi_1 = \frac{2\omega n_1}{\omega^2 - d}, \quad \operatorname{tg}\varphi_2 = \frac{2n_1 q}{\omega^2 - d + 2n_1^2}\end{gathered} \tag{22}$$

The oscillations determined by the formula (22) become resonance oscillations for $\omega_p = (d - 2n_1^2)^{1/2}$. The resonance amplitude is $N_p = (2n_1 q)^{-1}$. With λ_2 increasing the values of ω_p and N_p decline. The damping coefficient which is determined by the value of n_1 is proportional to λ_2. It is also noticed that for λ_2 increasing the phase shift grows between the chamber oscillations and the pressure at infinity. For $\lambda_2 = 0$ the formulas (22) describe the small oscillations of a gas-filled chamber in an incompressible elastic body.

In the case of Q = 0 one finds from Eq. (16) that for a Voigt medium

$$\begin{gathered}\Delta R = \gamma(\omega^2 + n_1^2)^{-1}[\omega^{-1}\sin(\omega t + \varphi_3) + te^{-n_1 t} + 2n_1(n_1^2 + \omega^2)^{-1}e^{-n_1 t}] \\ \operatorname{tg}\varphi_3 = 2\omega n_1(\omega^2 - n_1^2)^{-1}\end{gathered} \tag{23}$$

The effect of λ_2 can be directly observed.

Finally, a formula for ΔR is now introduced for a Voigt medium in the case of Q < 0:

$$\Delta R = \gamma\omega^{-1}N\sin(\omega t + \varphi_1) + {}^1/_2\gamma(n_1^2 - d)^{-1/2}[(n_3^2 + \omega^2)^{-1}e^{-n_3 t} - (n_2^2 + \omega^2)^{-1}e^{-n_2 t}], \quad n_{2,3} = n_1 \pm (n_1^2 - d)^{1/2} \tag{24}$$

In conclusion, it should be pointed out that one should set $\sigma = 0$ when using the relations (22)-(24).

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